

Title: Exploring Patterns

Brief Overview:

During five to seven math sessions, students will identify, create, and extend patterns. They will identify pattern rules. Students will be assessed on their problem solving abilities based on pattern skills taught during the unit. They also will enter a contest where they must create an original pattern and write a persuasive letter.

NCTM 2000 Principles for School Mathematics:

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*
- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*
- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*
- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*
- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*
- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

Links to NCTM 2000 Standards:

- **Content Standards**

- **Number and Operations**

- *Understand numbers, ways of representing numbers, relationships among numbers, and number systems.*
 - *Understand meanings of operations and how they relate to one another.*
 - *Compute fluently and make reasonable estimates.*

- **Algebra**

- *Understand patterns, relations, and functions.*
 - *Represent and analyze mathematical situations and structures using algebraic symbols.*
 - *Use mathematical models to represent and understand quantitative relationships.*
 - *Analyze change in various contexts.*

- **Geometry**

- *Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.*
 - *Use visualization, spatial reasoning, and geometric modeling to solve problems.*

- **Process Standards**

- **Problem Solving**

- *Build new mathematical knowledge through problem solving.*
 - *Solve problems that arise in mathematics and in other contexts.*
 - *Apply and adapt a variety of appropriate strategies to solve problems.*
 - *Monitor and reflect on the process of mathematical problem solving.*

- **Reasoning and Proof**

- *Recognize reasoning and proof as fundamental aspects of mathematics.*
 - *Make and investigate mathematical conjectures.*
 - *Develop and evaluate mathematical arguments and proofs.*
 - *Select and use various types of reasoning and methods of proof.*

- **Communication**

- *Organize and consolidate their mathematical thinking through communication.*
 - *Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.*
 - *Analyze and evaluate the mathematical thinking and strategies of others.*
 - *Use the language of mathematics to express mathematical ideas precisely.*

- **Connections**

- *Recognize and use connections among mathematical ideas.*
 - *Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.*
 - *Recognize and apply mathematics in contexts outside of mathematics.*

- **Representation**

- *Create and use representations to organize, record, and communicate mathematical ideas.*
 - *Select, apply, and translate among mathematical representations to solve problems.*
 - *Use representations to model and interpret physical, social, and mathematical phenomena.*

Links to National Science Education Standards:

- **Physical Science**

- The students will be given the opportunity in the extension activities to create a structure using toothpicks, incorporating a pattern, and testing structural integrity.

- **Life Science**

- The students will be given the opportunity in the extension activities to observe the school and surrounding area for patterns in the school and in nature.

Grade/Level:

Grades 3-4

Duration/Length:

This unit will last approximately five periods. Each lesson will take approximately 45 minutes to one hour.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Basic pattern recognition
- Persuasive letter writing
- Basic shape recognition

Student Outcomes:

Students will be able to:

- identify patterns and pattern characteristics (pattern cores, etc.).
- extend and continue patterns.
- name pattern rules and relationships.
- create and design a pattern.

Materials/Resources/Printed Materials:

- Markers, pencils, crayons, glue, stickers, toothpicks
- Pattern blocks
- Blank sentence strips
- Index cards
- Writing paper
- Teacher Resource Sheets #1-8
- Student Resource Sheets #1-10
- Additional resource books:
 - Beginning Algebra Thinking, Grades 3-4 @ Ideal School Supply Company: 1994; ISBN: 1-56451-095-6
 - Math to Know @ Houghton Mifflin Company Great Source Education: 2000; ISBN: 0-669-47153-4(softcover)
 - 20 Thinking Questions for Pattern Blocks, Grades 1-3 @ Creative Publications: 1995; ISBN: 1-56107-790-9
 - Cooperative Problem Solving With Pattern Blocks @ Creative Publications: 1989; ISBN: 0-88488-731-6
- Web site: <http://forum.swarthmore.edu/dr.math>

Development/Procedures:

Lesson 1: Pattern Basics

- *Motivation:* Demonstrate patterns using students themselves as models. Bring the class into a circle and create simple repeating patterns. (Ex. boy, girl, boy, girl; brown hair, blonde hair, black hair, brown hair, blonde hair, black hair) Refer to Teacher Resource Sheet #1.
- Have the students explain patterns through writing a description of patterns in their math journals.
- Model patterns using pattern blocks. Begin with a simple pattern. (Ex. ABABAB...). Have the students copy and continue the pattern for 3 more terms.
- Have a class discussion on the characteristics of the patterns you have modeled. Refer to Teacher Resource Sheet #1-2.
- Model a new pattern following previous guidelines. (Ex. AABAAB..., ABAC... etc.) Continue with discussions of that pattern's characteristics.
- Repeat with other patterns and variety of manipulatives. (Ex. toothpicks, unit cubes, etc.)

- Have students create a new pattern with a cooperative group containing 2-4 students. Exchange group patterns. Have the groups copy, extend, and describe the new pattern.

Lesson 2: Creating Basic Patterns and Connecting Similar Patterns and Problems

- *Motivation:* Give each group a sentence strip containing a pattern made from pattern blocks, followed by 3 blanks. Have the students think-pair-share what will come next in the pattern. Have each pair copy, extend, and describe the pattern using pattern blocks. Refer to Teacher Resource Sheet #3.
- Instruct each pair to identify the core of the pattern. Have them illustrate and describe the core in their math journals.
- Have students exchange the sentence strips with another group and repeat the copy, extend, and describe process. Once again, students should illustrate and describe the core in their math journals.
- Model an AB, AB, AB, pattern with toothpicks on the overhead for the students, Teacher Resource Sheet #3. Ask students if they have seen the pattern before. Allow them think-pair-share. Share responses from the discussion and as a class, translate to a letter pattern.
- Give pairs of students toothpicks to copy, extend, and describe a different pattern on the overhead. Ask pairs to identify by copying it using letter representation.
- Provide each student with a blank sentence strip and access to pattern blocks, stamps, or stickers. Have the students construct a pattern first using the pattern blocks. Allow students to copy the pattern on the sentence strip with the stickers or stamps. Students must write the rule on the back of the strip. Collect and inform the students that these will be used as the pattern of the day.

Lesson 3: Patterns with Charts and Tables (This lesson can take 2-3 class periods.)

- *Motivation:* Select a student to post one sentence strip pattern from Lesson 2 as the “Problem of the Day”. Have students copy, extend, and describe the pattern in their journal. The student who chose the sentence strip does not solve, but leads the discussion following the journal writing. Hint: The sentence strip can be a new daily starter activity to involve all students in the learning process.
- Provide students with pattern blocks, scrap paper, etc. On the overhead, display “crab” model found on Teacher Resource Sheet #4A.
Present the following directions:
 - One crab takes 3 blocks. Have the students copy with their blocks.
 - Two crabs take 6 blocks. Have the students copy with their blocks.
 - Inform the students to create three crabs. Ask the students how many blocks will it take to make three crabs.
 - How many blocks will be needed for 10 crabs? Give time for students to solve on their own.
 - Monitor students, checking for understanding.
 - Discuss strategies used in reaching their solution (If no one in the class created a table/chart to solve, ask, “What are some other ways/strategies we could use to organize our information?”)
- Model the chart from Teacher Resource Sheet #4B on the overhead. Have students instruct you as to how to complete the table/chart.
- Invite students to name and summarize the relationship between the number of crabs and the number of blocks used. (Rule is 3 times number of crabs=number of blocks.)

- Display Teacher Resource Sheet #5. Inform students that they will now be looking at the crab in a different way. Present the following directions:
 - One crab has 2 claws. Have the students copy with their blocks.
 - Two crabs have 4 claws. Have the students copy with their blocks.
 - Inform the students to create three crabs. How many claws do 3 crabs have?
 - How many claws do 12 crabs have? Give students time to solve independently.
- Monitor students by checking for strategies used.
- Discuss strategies used allowing the students time to explain their chosen strategy. List all answers given on the overhead/chalkboard if the students could justify their response.
- Model the chart from Teacher Resource Sheet #5 on the overhead. Have students advise you as to how to complete the table/chart.
- Invite students to name and summarize the relationship between the number of crabs and the number of claws. If the students do not point it out, show them the 2 times table relationship.
- Display Teacher Resource Sheet #6 on the overhead. Allow students to read the problem themselves. Give the students time to copy, extend, and describe the pattern.
- Discuss strategies used to find patterns and have students explain the strategies they used.
- Have students complete Student Resource Sheet #2.

Lesson 4- Reading Directions to Perform a Task

- *Motivation:* Create cooperative groups with the following activity.
 1. For a group of 20 students, supply the following pattern blocks:
 - 1 tray of 5 triangles
 - 1 tray of 5 squares
 - 1 tray of 10 trapezoids
 2. At the beginning of the session, have each child choose a block.
 3. Have the ABAC core pattern on the board.
 4. Instruct the students to utilize the blocks in their hands to join 3 other students creating the core pattern on the board.
 5. Inform the students that they have now created the groups they will be working with for the remainder of the lesson.
- Use Teacher Resource Sheet #7 for directions for completing the 4-step direction activity.
- Provide the groups with a set of four step directions found on Student Resource Sheets #3-8. Instruct the students to each take one of the steps and read and share with the group.
- Instruct the students to create the pattern on their direction cards. All four steps are needed to construct the pattern.
- Monitor groups to ensure every student is actively involved in the activity. When pattern is completed, give the group a second set of directions. (Make sure all groups complete at least two sets of direction cards.)
- Instruct all groups that they will be creating their own four-step direction cards.
- Provide each group with four index cards to record their directions. Allow the groups ample amount of time to complete this activity.
- Use these student created four-step directions as:
 - a group exchange
 - a center activity
 - a transition activity

Lesson 5: Culminating Activity

- *Motivation:* Read the vignette Student Resource Sheet #9 with the students. Provide the students with the self-assessment Student Resource Sheet #10. Discuss the expectations and go over the rubric.
- Provide pattern blocks, toothpicks, sentence strips, etc. for student use.

- Have the students create a pattern for the art contest.
- Provide the students with paper to write their persuasive letter.
- Provide the students with time to complete the Student Resource Sheet #10 .

Performance Assessment:

- The art teacher is having a contest. The students will need to create an original pattern where they need to include a core and a rule to demonstrate attainment of the concept.
- See Student Resource Sheet #10 and Teacher Resource Sheet #8.

Extension/Follow Up:

Science

- Have students use toothpicks to create a structure that incorporates a pattern.
- Have students observe the school and the surrounding area to find patterns in the school and in nature.

Math.

- Have the students place their patterns in a display center or hallway.
- Have students write word problems relating to lessons in the pattern unit.

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Questions For Problem Solving

How did you solve the problem?

Why did you solve the problem that way?

Is there another solution?

Is there another way to solve the problem?

What strategy(ies) did you use?

Explain why you think your answer is right?

Do you see a pattern?

Problem Solving Strategies

Patterns

Make/use a model

Make/use a picture

Logical deduction

Guess and test

Make/use a table

Make/use an organized list

Work backwards

Solve a simpler problem

Make/use a diagram(picture)

Patterns:

General rule

Particular

Write an equation

Trial and error(Guess and test)

Use a model(Simulation)

Inductive reasoning

Deductive reasoning

Vocabulary

core - The core of a repeating pattern is the shortest string of elements that repeat.

function - A set of ordered pairs such that for any first number (the input), there is only one possible second number (the output).

pattern - A pattern is a sequence of objects, numbers, color, etc. that repeats. The core should extend three times when copied by a student.

relationship - A relationship is a description of how a set of ordered pairs relate.

rule - A rule is a description of the pattern.

sequence - A set of numbers arranged in a special order or pattern.

term - A term is each place or position in the sequence.

Basic Pattern Sequences

AB, AB, AB, AB, AB,...

AAB, AAB, AAB, ...

ABAC, ABAC, ABAC,...

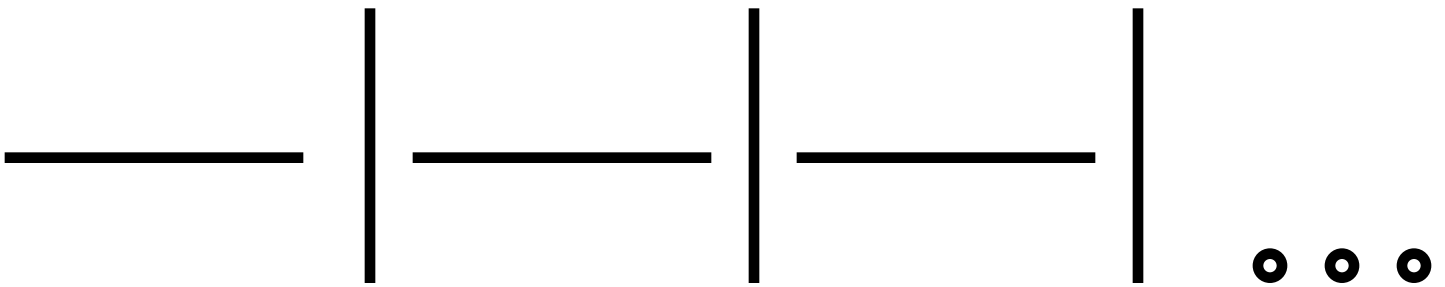
ABC, ABC, ABC, ...

AABBCC, AABBCC, AABBCC, ...

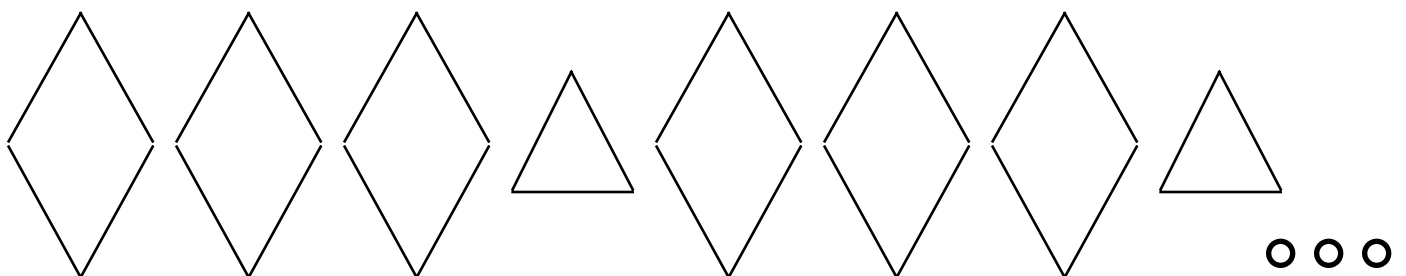
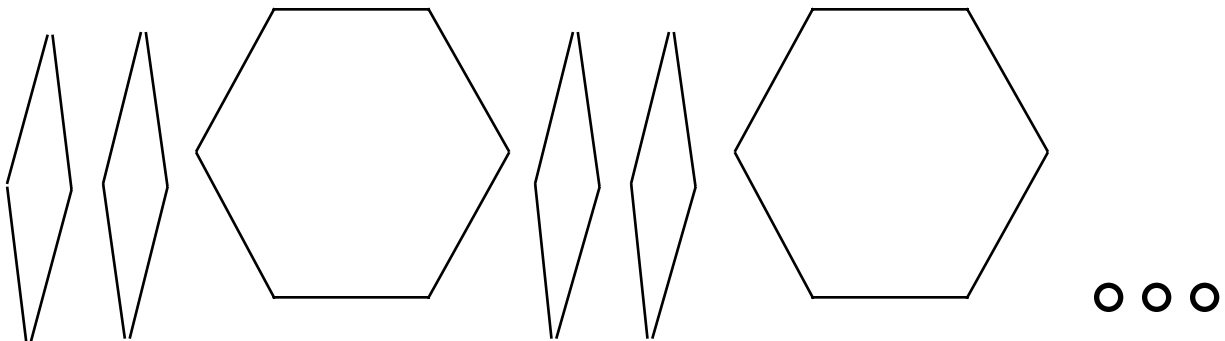
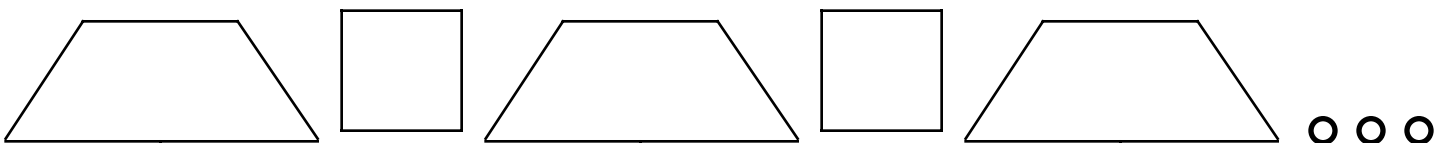
AABB, AABB, AABB, AABB, ...

ABA, ABBA, ABBBA, ABBBBA, ...(A Growing Pattern)

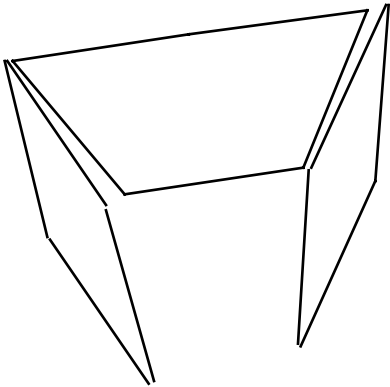
Toothpick Overhead Activity



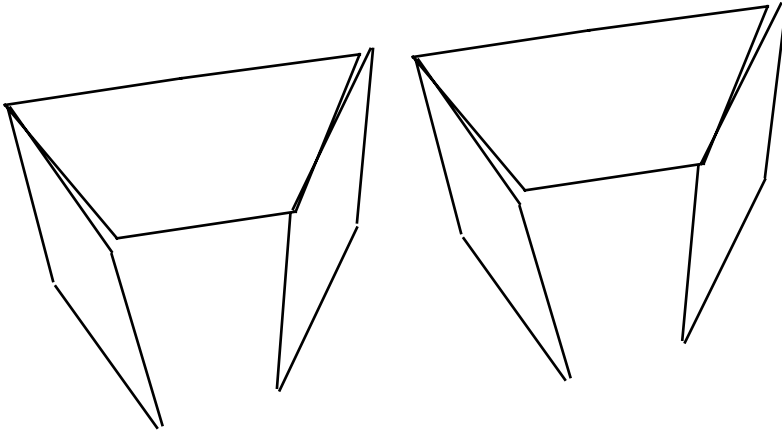
Pattern Blocks Sentence Strip Examples



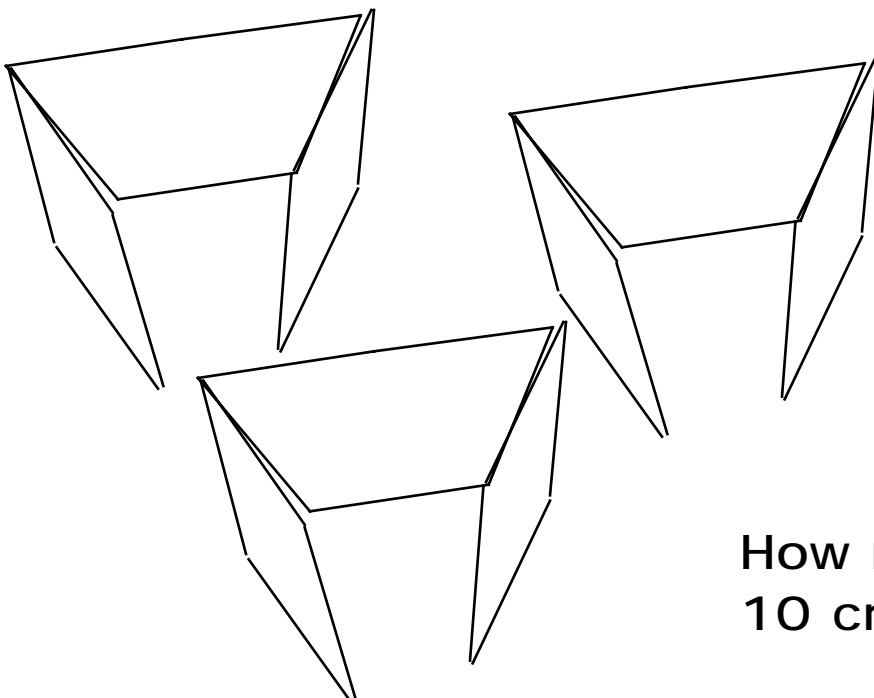
Crabby Blocks



1 crab takes
3 blocks.



2 crabs take
6 blocks.



3 crabs take
_____ blocks.

How many blocks would
10 crabs take?

Example Chart For Crabby Blocks

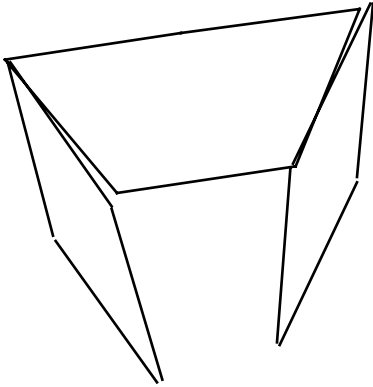
# of Crabs	# of Blocks
1	3
2	6
3	9
10	30

What is the relationship between the two columns?

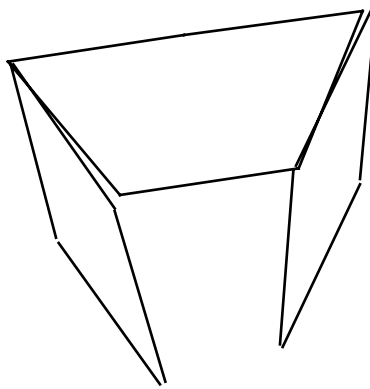
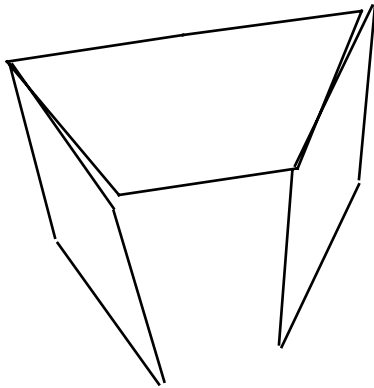
Answer: Take the # of crabs and multiply it by 3 to find the # of blocks.

**With C = # of Crabs and B = # of Blocks, then
 $C \times 3 = B$**

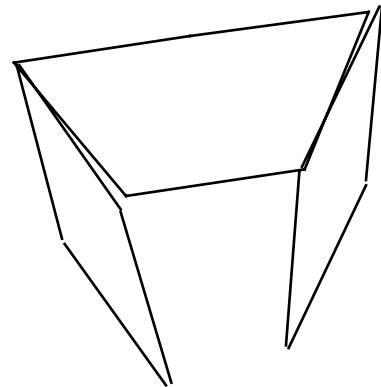
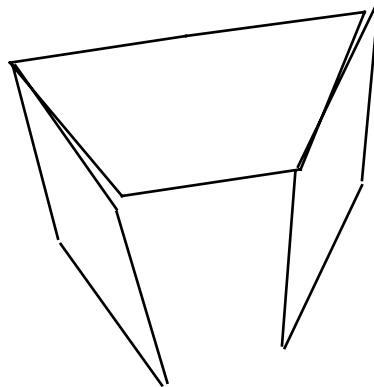
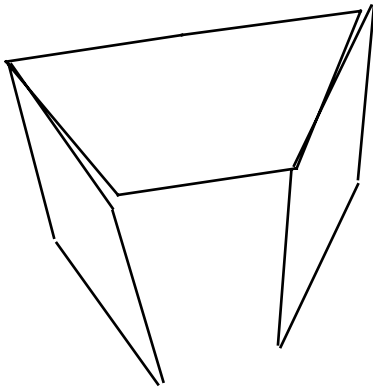
Claws of Crabs



1 crab has
2 claws.



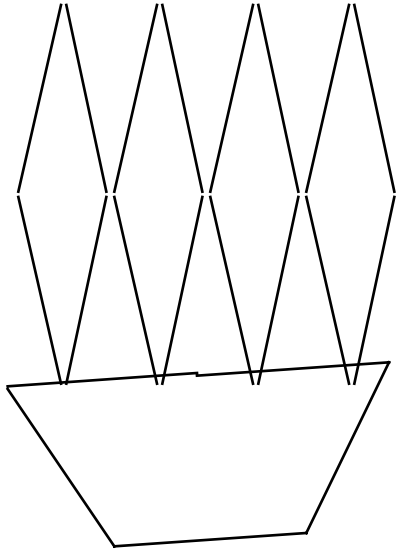
2 crabs have
4 claws.



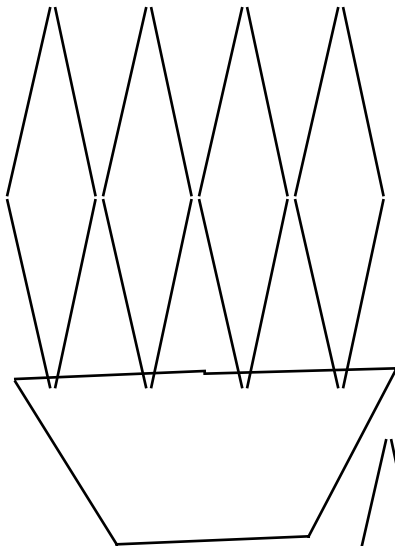
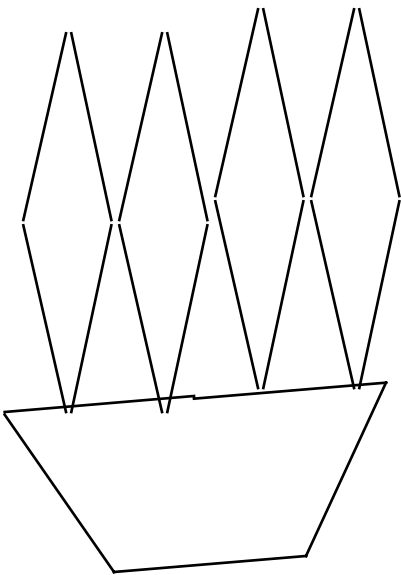
3 crabs have
_____ claws.

How many claws do 12 crabs have?

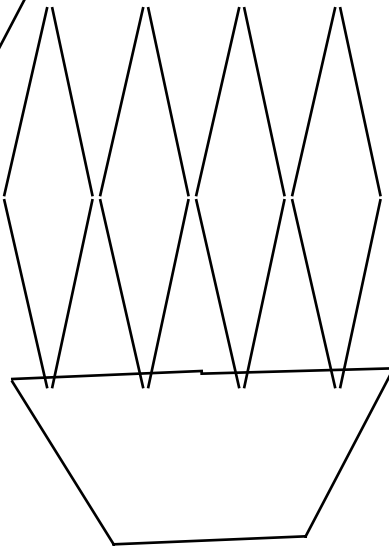
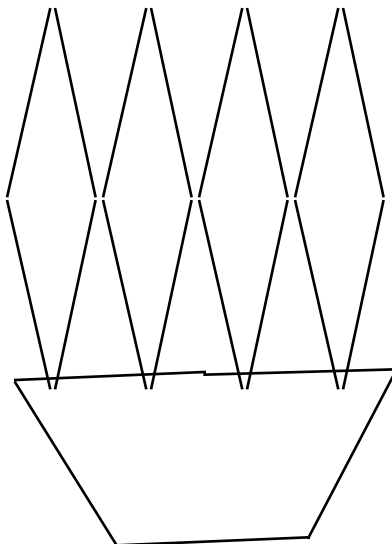
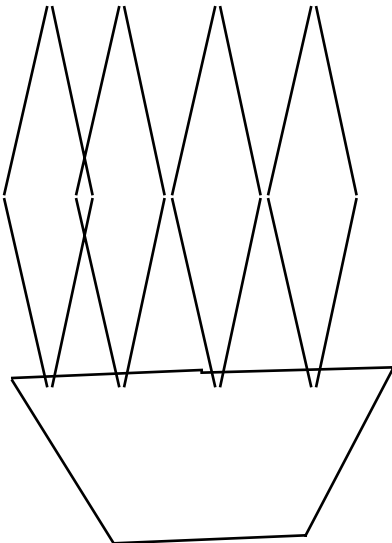
Tall Ships Are Sailing



1 tall ship has
4 sails.



2 tall ships
have 8 sails.



3 tall ships
have ____ sails.

How many sails do 11 tall ships have?

Direction Sheet for Following Directions Activity

Pictures on the problem sheets can be used as a way to keep each problem organized. You also can use them to divide by groups. The pictures are not the pattern that will be made.

1. Cut apart each problem sheet.
2. Give each group one set of cut out directions. Each problem is represented by one set of pictures.
3. Have each student take one card. If there are less than 4 students in a group, have someone take 2 cards.
4. Have each student silently read the card and then share information on the card with the group.
5. Students need to design the pattern indicated on their cards.
6. As they finish, give students a second set to complete in the same way.
7. Each group should do at least 2 sets of directions.

Pattern Project: Art Contest
Scoring Key

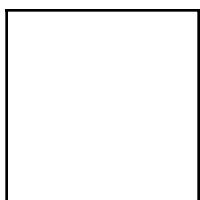
- 4** **Pattern is accurately represented. Pattern is clear and repeating at least 5 times. Rule and core of pattern are stated and accurate. All parts of a persuasive letter are included. Proper use of math terms.**

- 3** **Pattern is accurately represented. Pattern is clear and repeating 3 times. Rule and core of pattern are accurately stated and accurate. One or two parts of a persuasive letter are missing. Proper use of math terms.**

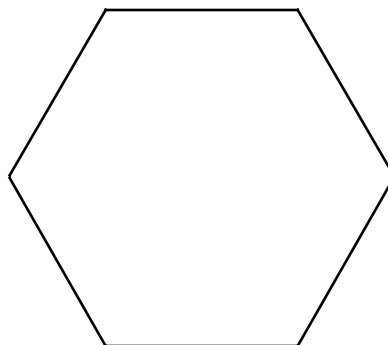
- 2** **Pattern is represented. Pattern is not clear and does not repeat. Rule and core of pattern are stated, but are inaccurate. More than 3 parts of a persuasive letter are missing. No math terms are used.**

- 1** **Project not completed.**

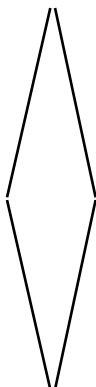
Pattern Blocks Template



square



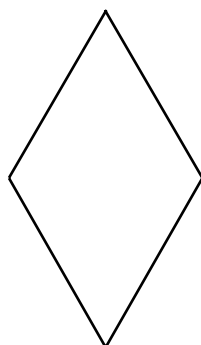
hexagon



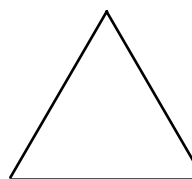
rhombus



trapezoid



parallelogram



triangle

Quiz: Word Problems

Problem 1: Show all work!

Word got around that "XYZ-Men" was a horrible movie! There were 70 people at the first show. Only 63 people came to see the second show, and 56 came to the third show, and 49 came to the fourth show. If the pattern continued, how many people came to see the ninth show?

Problem 2:

On Monday, 9 turtles are in the Ninja Turtle Club on the first day. Fifteen turtles are present on the second day, 21 turtles are present on the third day and 27 on the fourth day. If the pattern continues, how many turtles will have joined the club on the tenth day?

Name _____ Date _____

Following Directions
Problem 1

Use the blocks to make a growing pattern.



Use tan blocks in your pattern.



Use more red trapezoids than any other shape.



Include green blocks in your pattern.



Following Directions
Problem 2

Start with 2 orange blocks.



Add 2 more blocks of different colors.



Create a repeating pattern.



Include a triangle in your pattern.



Following Directions
Problem 3

Start with a red block.



Create a repeating pattern.



Add 2 more blocks,
both the same color.



Do not use any shapes
that have 4 sides.



Following Directions

Problem 4

Make a line of blocks in a repeating pattern that curves across the table.



Start with 1 red block.



Use 4 different colors of blocks in the pattern.



The rhombus and parallelogram are next to each other.



Following Directions

Problem 5

Make a line of blocks in a repeating pattern.



Use 2 different colors of blocks.



Use an orange block for 1 color.



Do not put 2 blocks of the same shape next to each other.



Following Directions

Problem 6

Make a line of blocks in a repeating pattern that curves across the table.



Start with a yellow hexagon.



Use 3 different colors of blocks in the pattern.



The blue and green blocks must be next to each other.



Pattern Project: Art Contest

The art teacher is having a contest. She wants all third and fourth graders students to participate in the competition. One requirement of the contest is that each work of art must have a pattern. Your task is to create an art project for the contest. Since we have just finished a unit on patterns make sure that the project includes a rule.

Secondly, write a friendly letter to the art teacher persuading her that your pattern is the best and why you should win the contest.

Name _____ Date _____

Self Assessment

1. I made a pattern.

yes ____ no ____

2. I can identify the core of my pattern.

yes ____ no ____

3. I can identify the pattern rule of my design.

yes ____ no ____

4. I repeated the core of my pattern in my design.

yes ____ no ____

Name _____ Date _____